Math 3236 Statistical Theory

4/13/23

X_i are Normal μ, σ^2

σ^2 Known.

Test on μ

$H_0 : \mu \leq \mu_0$

$H_a : \mu > \mu_0$

$i = 1 \ldots N.$

$$\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i$$

$\bar{X} > c$ reject

$c > \mu_0$

$$z = \sqrt{N}\left(\frac{\bar{X} - \mu_0}{\sigma}\right)$$

$$\bar{X} \geq \mu_0 + \sigma c / \sqrt{N}$$

$$Z \geq c$$

Test of size α.

$$\pi(\delta \mid \mu) = \mathbb{P}\left(\bar{X} \geq \mu_0 + \sigma c / \sqrt{N} \,\middle|\, \mu\right)$$

if μ is the correct value

$$\sqrt{N}\left(\frac{\bar{X} - \mu}{\sigma}\right) \simeq N(0,1)$$

$$= \mathbb{P}\left(\left(\frac{\bar{X} - \mu}{\sigma}\right)\sqrt{N} \geq \left(\frac{\mu_0 - \mu}{\sigma}\right)\sqrt{N} + c \,\middle|\, \mu\right)$$

$$= 1 - \Phi\left(\left(\frac{\mu_0 - \mu}{\sigma}\right)\sqrt{N} + c\right) = \pi(\delta \mid \mu)$$

$$\sup_{\mu \leq \mu_0} \pi(\delta \mid \mu) = \pi(\delta, \mu_0)$$

The condition for δ to be a size α Test

$$\sup_{\mu \leq \mu_0} \pi(\delta \mid \mu) = \alpha$$

$$\Downarrow$$

$$\pi(\delta \mid \mu_0) = \alpha$$

$$\alpha = 1 - \Phi(c)$$

if $z > \Phi^{-1}(1-\alpha)$ reject H_0

if $z \leq \Phi^{-1}(1-\alpha)$ do not reject

if $\bar{X} > \mu_0 + \sigma \Phi^{-1}(1-\alpha)/\sqrt{N}$ reject

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o

$\bar{x}$ for very small α
do not reject

$\bar{x}$ is large, I do not
reject.

For any $\bar{x}$ There is a
value $\bar{\alpha}$ of α such That
I reject if $\alpha > \bar{\alpha}$
do not reject if $\alpha < \bar{\alpha}$

$\bar{\alpha}$ is The p-value of
The Test.

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p-value is 0.01

z is The result

p value is

$1 - \Phi(z_0)$

$z \geqslant \Phi^{-1}(1-\alpha)$ reject

reject if $\Phi(z) \geqslant 1-\alpha$

reject if $\alpha \geqslant \underbrace{1 - \Phi(z)}_{\text{p-value}}$

If you have a Test δ_t $T \geq t$ reject. If the observed value is t_0

p-value $\sup_{\theta \in \Omega_0} \pi(\theta \mid \delta_{t_0}) = \sup_{\theta \in \Omega_0} \mathbb{P}(T > t_0 \mid \theta)$

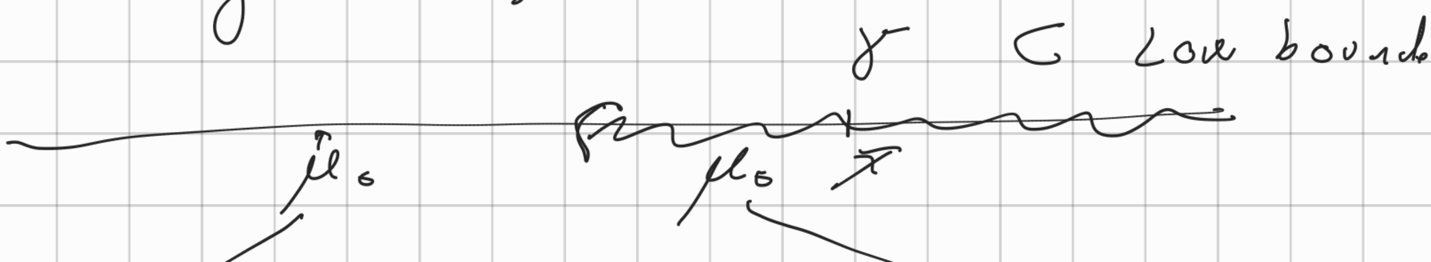
if I saw $\bar{x}$

$$\bar{x} - \Phi^{-1}(1-\alpha)\,\sigma/\sqrt{N} > \mu_0$$

reject.

$\bar{x} - \Phi^{-1}(1-\alpha)\,\sigma/\sqrt{N}$ is a coeff $1-\alpha$ confidence lower bound on μ

I got $\bar{x}$



reject H_0 $\mu \leq \mu_0$

do not reject H_0